The following equations and the attached programs are described in Filament Winding, Composite Structure Fabrication pp5-43-50, 2nd Ed., 1990, © SAMPE Publishers

Netting analysis is a simple procedure for predicting stresses in a fiber-reinforced composite by neglecting the contribution of the resin system. The procedure has intuitive appeal, and it gained acceptance early in the development of composite structures. The technique applies static equilibrium principles with no consideration given to strain compatibility. Even with such theoretical flaws, netting analysis continues to survive, and is sometimes useful for sizing and for predicting failure in simple structures. However the technique should not be extended to other than simple pressure vessels and the design of any composite structure must be based on a thorough understanding of laminate plate theory

As an example application, consider a filament wound cylinder of radius R pressurized with an internal pressure $\mathbf{p_i}$

If the vessel is wound with only helical **(±θ)** fibers, with an allowable fiber stress $\sigma_{f\theta}$ determine the helical fiber thickness $(\mathbf{t}_{f\theta})$ and the wind angle θ.

Figure 5.11 shows forces acting on a two-ply helical layer with a cut of unit width (a) in the axial plane, and (b) in the circumferential plane. Summing forces in the axial **(m)** direction in Figure 5-11a shows:

$$N_m = \sigma_{f\theta} t_{f\theta} \cos^2\theta = \frac{P_i R}{2} \qquad \textbf{[5.61]}$$

Solving Equation [5.61], the helical fiber thickness required to contain the internal pressure is:

$$t_{f\theta} = \frac{P_i R}{2\sigma_{f\theta}\cos^2\theta} \qquad \textbf{[5.62]}$$

Summing forces in the circumferential (h) direction in Figure 5.12b, produces:

$$N_h = \sigma_{f\theta} t_{f\theta} \sin^2\theta = P_i R \qquad \textbf{[5.63]}$$

Using $\mathbf{t}_{f\theta}$ from Equation [5.62] in [5.63] shows that $\mathbf{tan}^2 \theta$ **=2**, or θ**=± 54.7** degrees. This is the wind angle required for a pressurized cylinder with helical windings only.

If the vessel is wound with both helical **(±θ)** fibers and hoop **(θ≈90°)** fibers, determine the helical and hoop fiber thickness $\mathbf{t}_{f\theta}$ and $\mathbf{t}_{\mathbf{f90}}$ respectively.

Figure 5-12 shows forces acting on a two-ply helical layer and a hoop ply with a cut of unit width, (a) in the axial plane, and (b) in the circumferential plane. Summing forces in the axial direction (Figure 5-12a) produces the helical fiber thickness required to contain the internal pressure, given by Equation [5.62]. Using this value for $\mathbf{t}_{f\theta}$ when summing forces in the hoop direction (Figure 5-12a) produces:

$$t_{f\theta} = \frac{P_i R}{2\sigma_{f90}}(2 - \tan^2 \theta) \qquad \textbf{[5.64]}$$

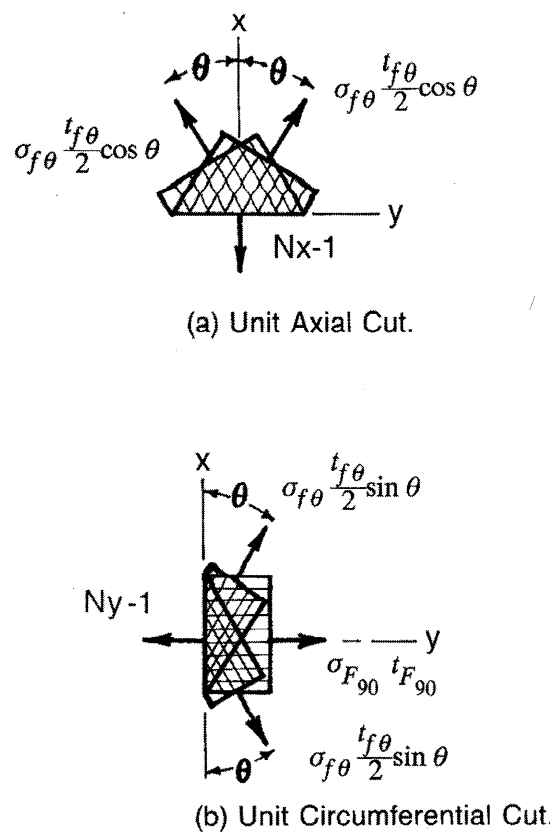

Figure 5-12.
Helical/hoop fiber element.

Where $\sigma_{\mathbf{f90}}$ is the fiber stress in the hoop fibers

Conversely, when the CSA value is not known (many fiber manufacturers may not include CSA in their product descriptions), it can be determined from the following [5.65a] from the density and yield of the fiber.

$$\frac{1}{\dfrac{density}{yield}} = CSA \qquad \textbf{[5.64a]}$$

In the winding process, N spools are used to form a winding band of width W. Each spool has a cross sectional area (CSA), values of which are listed in Table 5.3 for several rovings.

The fiber thickness for each ply as wound is:

$$t_{fP} = \frac{N(CSA)}{W} \qquad \textbf{[5.65]}$$

The fiber stress at failure in a composite laminate is less than that for a unidirectional coupon test or a strand tensile test. This reduction is called the translation efficiency; typical values are 70 to 80% of the strand tensile value. For example, the allowable stress for Kevlar® 49 fibers is 3800 MPa (550 ksi) in a strand tensile test (Table 2-2). A high value of fiber stresses for Kevlar® 49 in a pressure vessel application is 2750 MPa (400 ksi), a translation efficiency of 73%. Translation efficiency is affected by fiber damage during wind, voids, complex stress states, etc., which are present in the laminate, and not in a strand tensile specimen.

Table 5-3 Cross Sectional Area of Typical Rovings

Fiber Type	CSA (mm^2)	CSA (in^2)
E-Glass type 30 (675 yield)	0.289	44.73×10^{-5}
S-2 glass (20 end)	0.268	41.52×10^{-5}
Kevlar® 49 (4 end)	0.352	54.55×10^{-5}
Thornel 300 12000 filament tow	0.466	72.20×10^{-5}
Type AS4 12000 filament tow	0.486	75.40×10^{-5}
Celion 12k 12000 filament tow	0.465	72.00×10^{-5}

In a biaxial tension field, like a pressure vessel, Kevlar® 49 typically exhibits lower translation efficiencies than glass or graphite. Kevlar®, with its fibrous microstructure, is easily split longitudinally by matrix cracks, if the matrix is tightly bonded to the fiber. If the fiber is coated with a release agent before winding, matrix cracking can bypass the fibers, improving efficiency. Usually, fibers where interlaminar shear strength is important would not be released. Reflecting this, in a cylindrical pressure vessel the hoop fiber would be released, while the helicals, with discontinuities at the dome cylinder junction and at the polar boss, would not be released. Translation efficiencies of the hoop fibers would typically be increased from 70 to 85% by releasing. Releasing glass and graphite fibers has been found to be ineffective in improving translation efficiencies for these fibers.

In a cylindrical pressure vessel, translation efficiencies are usually lower for helical fibers than for hoop fibers. This is because the helical patterns include more overlaps and crossovers, as well as discontinuities at the dome cylinder junction and in the polar boss region. For these reasons, the helical-to-hoop fiber stress ratio used in design is usually between 75 and 100%.

In winding, the number of plies is selected so that the ply thickness in Equation [5.65] divides equally into the total fiber thickness required, Equations [5.62] and [5.64]. Band widths are selected in conjunction with the number of spools so that a band thickness (including resin) of about 100 to 250 μ (0.004 to 0.010 inches) is produced. This provides a band that is neither too thick nor too thin. For the helical windings, the band width W_θ is selected to cover the cylinder with no overlap with a whole number of bands wound at the angle θ. The helical ply thickness (including resin) is then:

$$t_{C_\theta} = \frac{N(CSA)}{W_\theta V_{f\theta}} \quad [5.66]$$

Where $V_{f\theta}$ is the fiber volume fraction for the helical windings. For the hoop windings, the band width W_{90} is selected so that the hoop windings do not deviate appreciably from the 90° wind angle. The hoop ply thickness (including resin) is then:

$$t_{C90} = \frac{N_{90}(CSA)}{W_{90} V_{f90}} \quad [5.67]$$

Where $\mathbf{V_{f90}}$ is the fiber volume fraction for the hoop plies. Table 5-4 provides typical ranges of fiber volume fractions attained with different fiber systems for helical and hoop plies. Actual fiber volume fraction depends on several process and geometric considerations including resin viscosity, mandrel diameter, winding tension, wind angle, processing time, B-stage temperature, and external pressure during cure. The fiber volume fraction is determined by using in-processing thickness measurements. Good process control produces repeatable dimensions.

Table 5-4 Typical Fiber Volume Fractions

Fiber	Helical	Hoop
Glass	0.55 - 0.60	0.65 - 0.70
Kevlar®	0.55 - 0.60	0.65 - 0.70
Graphite	0.50 - 0.55	0.60 - 0.65

Numerical Example

The above equations are used to provide preliminary thickness for the standard ASTM D-2528 pressure vessel (Figure 5-13) designed for a 45.5 MPa (6600 psi) burst pressure and an inside diameter of 146 mm (5.75 inch). If the winding machine is set up for a 5.8 mm (0.23 inch) hoop band width and 5.1 mm (0.2 inch) helical band width, the specified helical wind angle θ in the cylinder is $\approx 12^\circ$. The vessel is to be wound with Kevlar® 49, 4-end aerograde roving (Table 2-2). Design allowables are chosen as 2.93 GPa (425 ksi) hoop fiber stress, and 2.21 GPa (320 ksi) helical fiber stress. This uses translation efficiencies of 77% for the hoops, and 58% in the helicals, with a 75% ratio of helical to hoop fiber stress.

The required helical fiber thickness Equation [5.62] is:

$$t_{f\theta} = \frac{P_i R}{2\sigma_{f\theta}\cos^2\theta} = \frac{45.5 \bullet 10^6 \bullet 73}{2 \bullet 2.21 \bullet 10^9 \cos^2 12} = 0.79mm(0.031inch)$$

Assuming 3 helical layers (6 helical plies), the calculated number of helical spools Equation [5.65] is:

$$N_\theta = \frac{t_{f\theta} W_\theta}{(CSA)} = \frac{0.79/6 \bullet 5.1}{0.352} = 1.9$$

The design would use N_θ= 2, the closest whole number. It may be desirable to repeat the calculations with a slightly modified band width so that the calculated number of spools is closer to a whole number:

The required hoop fiber thickness Equation [5.64] is:

$$t_{f\theta} = \frac{P_i R}{2\sigma_{f90}}(2 - \tan^2\theta) = \frac{45.5 \bullet 10^6 \bullet 73 \bullet 92 - \tan^2 12}{2 \bullet 2.93 \bullet 10^9} = 1.11cm(0.0436in)$$

Assuming 9 hoop plies, the number of hoop spools Equation [5.66] is:

$$N_{90} = \frac{t_{f90} W_{90}}{CSA} = \frac{1.11/9 \bullet 5.8}{0.352} = 2$$

Assuming a fiber volume fraction of 0.6 for the helical, and 0.65 for the hoops (Table 5-5), the total composite thickness is Equations [5.66 plus 5.67]:

$$t_c = \frac{0.79}{0.6} + \frac{1.11}{065} = 3.0mm(0.12inch)$$